AUTOMATIC TIME MARK GENERATION ON THE TR-48®/DES-30 DESK-TOP HYBRID COMPUTING SYSTEM

by
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INTRODUCTION

This study describes a hybrid analog-digital program which, by utilizing digital timing and logical control signals, automatically marks time or frequency intervals on parametric plots, such as phase-plane or Nyquist diagrams, generated by an analog computer. The program is implemented on the TR 48 General Purpose Analog Computer with the DES 30 Digital Expansion System. The DES 30 is a low-cost, general-purpose digital logic package which, although capable of autonomous operation for digital logic design or instruction, is intended to operate primarily with desk-top analog computers. With an analog computer, such as TR 48, the DES 30 provides basic hybrid capabilities to the small computer facility.

In parametric plots, the variables which are applied to the X and Y axes of the plotter are both functions of a third variable, the parameter. The parameter does not appear explicitly on the resulting plots, so that often their interpretation is complicated. The object of the program is to alleviate this difficulty by automatically marking segments, which correspond to known intervals of the parameter, along the solution curves (trajectories).

Phase-plane diagrams are typical of type of plots being considered. Phase-plane diagrams display trajectories of dx/dt vs. x, where x may be the displacement and dx/dt the velocity of a moving body. Both the displacement and the velocity are functions of time t, the parameter. The velocity of a point along the phase-plane trajectory is the phase velocity, $ds/dt = \sqrt{\dot{x}^2 + \dot{y}^2}$ (not to be confused with the velocity of the body, dx/dt). Points at which $ds/dt = 0$ are termed singular points; at each such point $dx/dt = dy/dt = 0$. The analysis of such phase-plane plots is simplified when marks, signifying equal time intervals, are placed on the trajectories.

THE PROGRAM

A simple scheme for generating such marks has been proposed(1). "Correction terms" are periodically superimposed on the x and y plotter inputs such that a short mark, perpendicular to the slope of y(x), is recorded. Between marks the plotter is permitted to proceed without interference. In mathematical notation, the plotter inputs are

$$\text{x axis:} \quad x(t) - \delta k \frac{dy}{dt}$$

$$\text{y axis:} \quad y(t) + \delta k \frac{dx}{dt}$$

where k is an arbitrary proportionality factor, and δ is an on-off (digital) function such that $\delta = 1$ for a short period R beginning at $t = \tau, 2\tau, 3\tau, \ldots n\tau \ldots$, and $\delta = 0$ otherwise. τ is the basic parameter interval; it is a constant for phase-plane diagrams, but may be variable for Nyquist plots. R is to be made as short as the dynamics of the available plotter will permit. The overall effect of the "correction terms", which are added every τ seconds to the plotter inputs, maintained for a period R, and then removed, is shown in Figure 1.

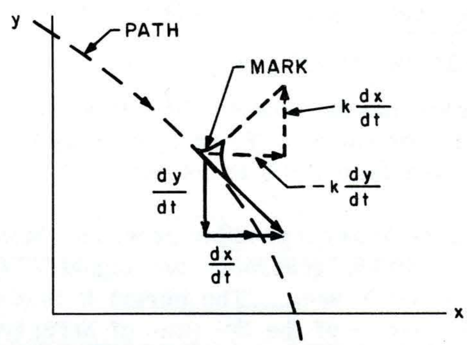

Figure 1. The Geometry of the Time Marks

Reference (1) Blache, Y.J.M.: Automatic Time Mark Generation for Analog Computer Phase Plane Trajectories; IRE Trans. on Electronic Computers, Vol. EC-11, #4, August 1962, pp. 571-572.

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The marks generated in this way are always perpendicular to the slope of the y(x) curve, and are proportional in magnitude to the phase velocity $ds/dt = \sqrt{\dot{x}^2 + \dot{y}^2}$. Thus, on phase-plane plots, the magnitude of the marks diminishes as the phase trajectories approach a singular point; this is a very desirable feature since near such points phase contours tend to become crowded. In addition, because of certain inherent properties of phase-plane contours, ($dx/dt \geq 0$ in quadrants I and II, limitations on the possible directions of contours, etc.), these marks are always on one side of the contour. There is no such guarantee for Nyquist plots.

In the case of Nyquist plots, the derivatives dx/dt and dy/dt must, in general, be generated. In the implementation discussed here, these derivatives are formed digitally, although this could be done equally well with well-known analog programs. In the case of phase-plane diagrams, dx/dt = y must be available, and dy/dt generally is, so the situation is much simpler. If dy/dt is not available, either analog or digital differentiation programs can be used.

IMPLEMENTATION WHEN DERIVATIVES MUST BE GENERATED

The time derivatives are computed approximately as follows. The time axis is divided into equal intervals T. At times t = T, 2T, 3T,...nT,... the value of the function whose derivative is desired (say x(t) to be concrete) is sampled and stored on an analog memory (track/store) unit.

The output of this track/store unit is termed -x(nT). At t = (n+1)T, -x(nT) is transferred to a second (cascaded) track/store unit, where it becomes +x(nT), and a new value, -x([n+1]T), is sampled and stored on the first track/store. Forming the difference of these two track/store outputs yields an increment in x:

$$-\Delta x = x(nT) - x([n+1]T)$$

and when this increment is divided by the constant time increment $\Delta t = T$, an approximation $-\Delta x/\Delta t$ to the derivative dx/dt is obtained.

Figure 2a shows a possible program. Monostable Timer 8 (MT8) generates the signal δ every 1 second ($\tau = 1$ sec). The period R is controlled by the setting of the ON time of MT8; typically, R = 100 milliseconds. When $\delta = 1$, a pair of digitally-controlled analog switches (DA switches E, F) conduct, thus adding the "correction terms" to the summing amplifiers whose outputs drive the arm and pen of the plotter.

Down counter 0 (DC 0) generates a carry-out (C_o) blip (a level which is high for one clock period only) every Δt seconds. As shown in Figure 2a, Δt = 20 milliseconds; however, in this program, Δt is adjustable from 1 to 100 milliseconds, since it is controlled by the setting of the preset-switch of DC 0, so that coarser or finer approximations of the derivatives can be computed (although finer Δt periods also result in increased noise sensitivity).

The carry-out blip of DC 0 triggers MT6, which generates a 100μs pulse. At the end of this 100μs pulse, MT6 triggers itself again via differentiator 7 and AND gate 13C.

The first 100μs pulse generated by MT6 is applied to track/stores B and D, causing them to TRACK for 100μs and return to STORE. The result of this operation is that the quantities $-kx(nT)/\Delta t$ and $+ky(nT)/\Delta t$, which were stored on track/stores A and C, are transferred to track/stores B and D, where these quantities become $+kx(nT)/\Delta t$ and $-ky(nT)/\Delta t$, respectively.

The second 100 μs pulse of MT6 is applied to track/stores A and C, causing them to sample and store new values, $-kx([n+1]T)/\Delta t$ and $+ky([n+1]T)/\Delta t$, respectively. At this time the summing junction (GJ) of DA switches E and F carry the signals $-k(\Delta x/\Delta t)$ and $+k(\Delta y/\Delta t)$ respectively. These are the proper "correction terms" except for the obvious sign inversion.

Figure 2b is the timing diagram for the program of Figure 2a. This diagram shows, incidentally, that for all practical purposes, AND gate 13B may be replaced by FF 23C.

No provision is made in this program for adjusting τ; this can easily be achieved by using counters to frequency-divide the 1 KC source. For Nyquist plots with variable τ (logarithmic frequency increments, for example), a variable-frequency pulse-generator may be used(2).

The function of the AND gate shown in broken lines in Figure 2a is to prevent DA switches E, F from conducting while the formation of the derivatives is in process. It may be removed at the cost of a "damaged" mark now and then.

Some saving in equipment can be effected with the alternative program shown in Figure 3. Here the function increment, Δf, for the approximation to

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Reference (2) DES 30 Reference Handbook, pub. #D800 2042 OA, EAI 1965, p. 42.

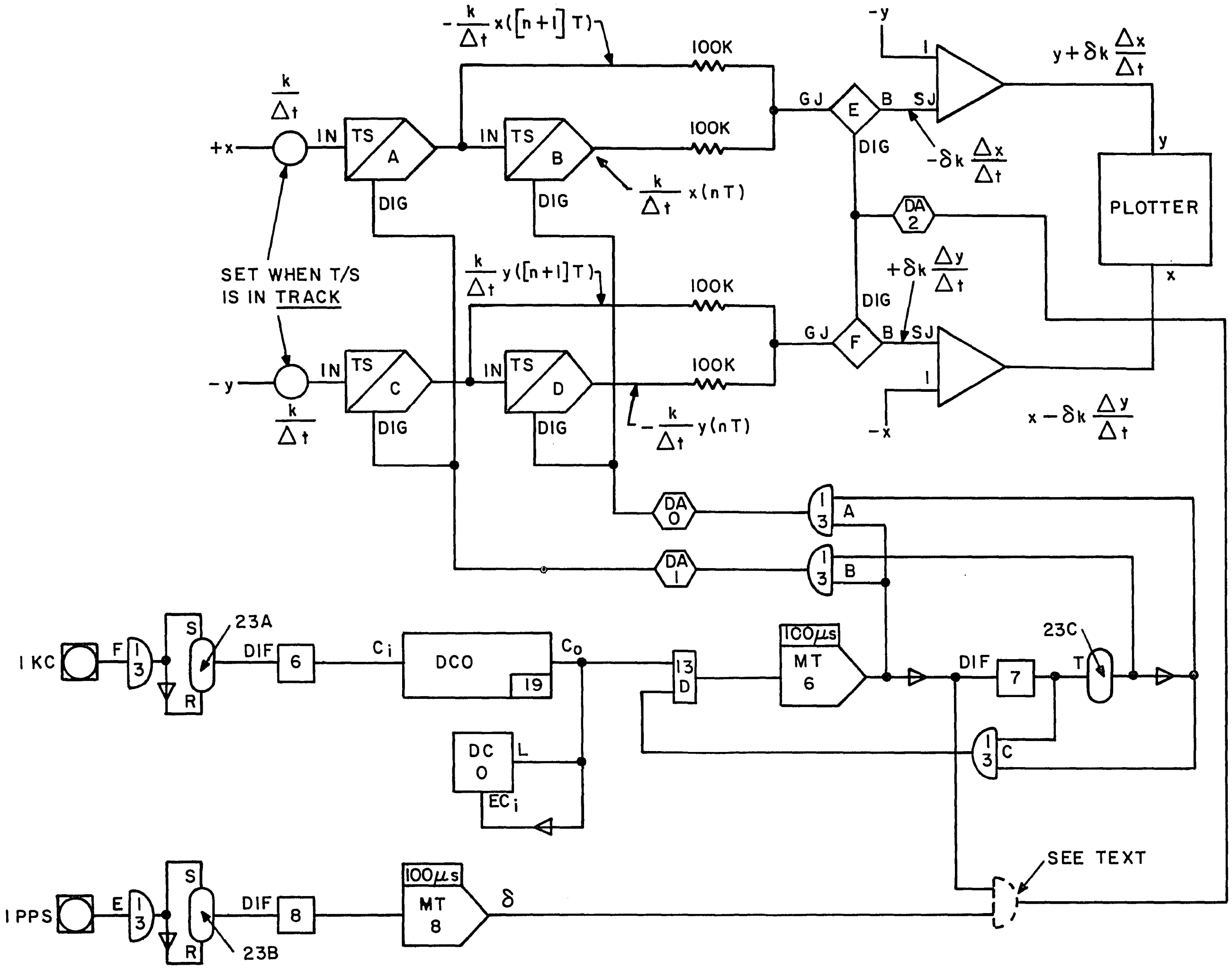

Figure 2a. Time Mark Program with Derivative Computation

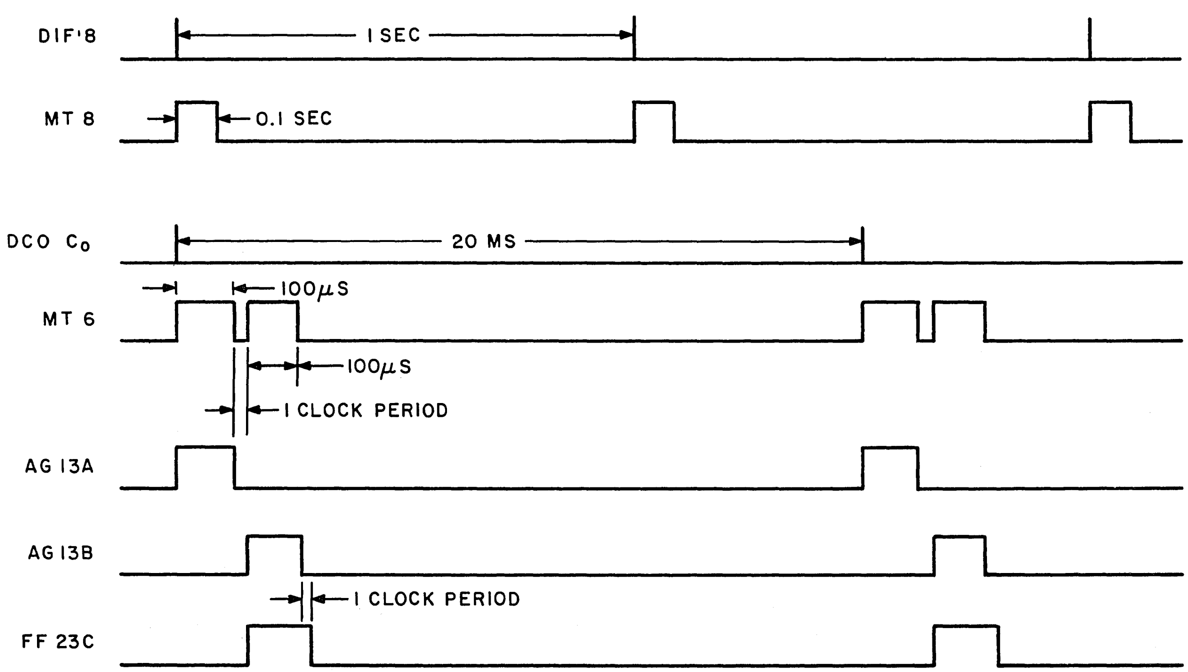

Figure 2b. Timing Diagram

the derivative, is formed by subtracting a sampled and stored past value of the function from the present value, but no attempt is made to store the present value. Thus the "present" value actually changes during the time that the mark is recorded, resulting in marks which are not so well-defined but are nevertheless quite acceptable for many applications. This program differs from the one in Figure 2a also in that derivatives are formed only prior to the generation of the mark, rather than continually. Here Δt is controlled by MT7 while MT8 generates the signal δ (controls R).

IMPLEMENTATION WHEN DERIVATIVES ARE AVAILABLE

In the case of phase plane plots, $dx/dt = y$ is available and so is, generally, dy/dt. Hence the program of Figure 4 suffices. Again, control over τ can be established with counters. The MT shown controls R.

PROGRAMMING INFORMATION

Attenuators which feed digital devices such as track/stores or DA switches must be set with the device in the ON (conducting, TRACK) state, since these devices present different loads in the ON and OFF states.

The program is initialized by setting the proper ON time controls of monostables and preset switches of down counters. Note that Δt is set on several attenuators, in addition to a monostable. Then clear the DES 30, place it in 1 MC RUN, and put the TR 48 from IC to OP.

The program of Figure 2a requires 3 Electronic Comparator 40.488 units on the TR 48, and 3 MT-DIF, 1 GPR, 1 AG trays on the DES 30. The program of Figure 3 requires 2 40.488 Comparator units on the TR 48 and the same complement of equipment on the DES 30 as in Figure 2a.

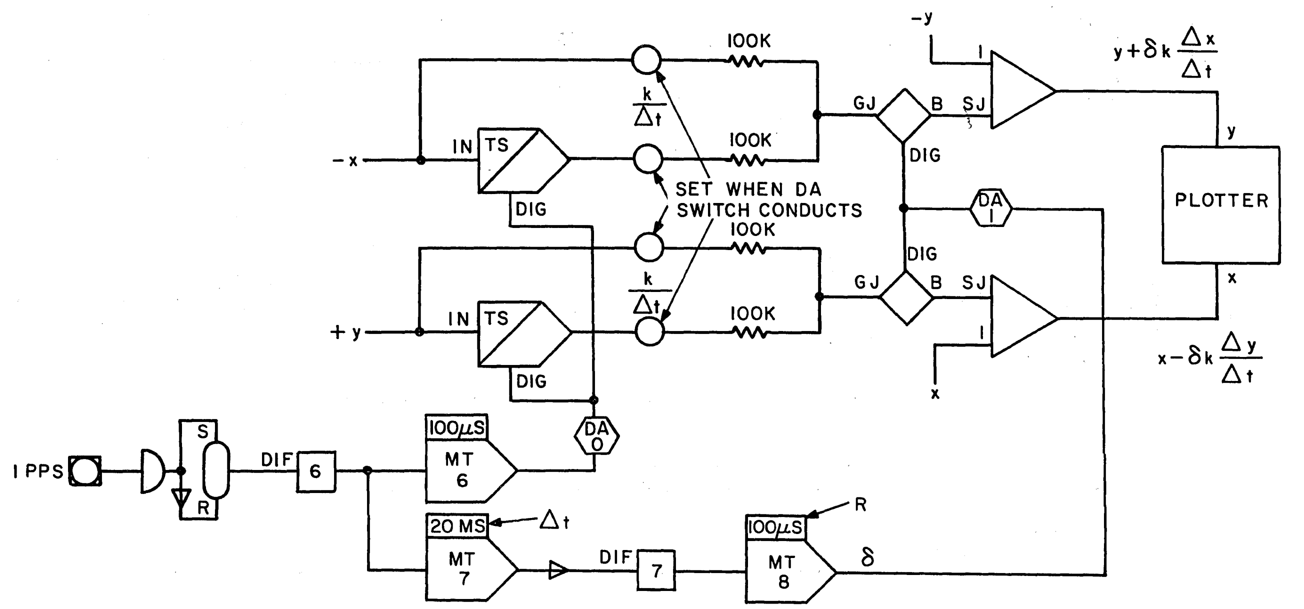

Figure 3. An Alternative Time Mark Program

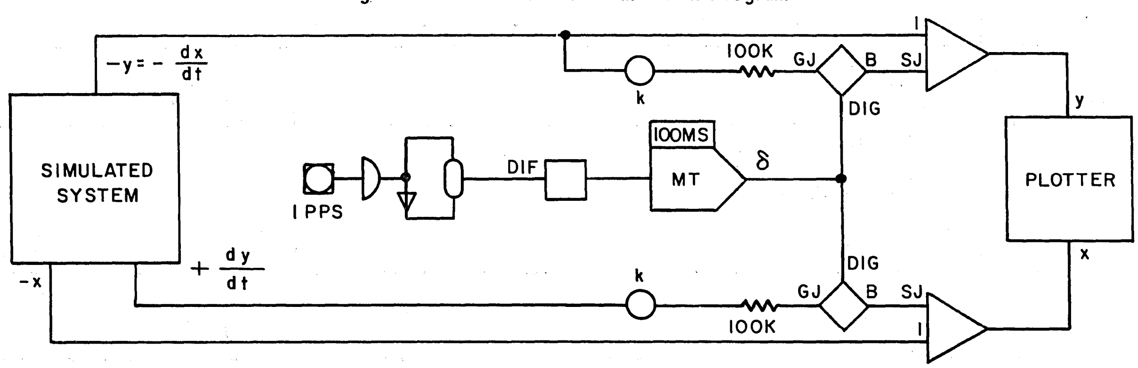

Figure 4. Time Marks for Phase Plane Diagrams