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Date: 28 July 1955

Subject: POLYNOMIAL FACTORIZATION

New Hitchcock's Method Routines

Two new routines for factoring higher order polynomials are described in this memorandum. Using 24,6 CS II code, the routines employ a method first described by Frank L. Hitchcock (Jour. Math. and Phys. 1944, p.69) and theoretically can handle polynomials with real coefficients up to 100th degree or higher, subject to certain restrictions.

Roots of the polynomial are printed out as quadratic factors, from which the roots are readily obtained by the quadratic formula. Roots may be real, imaginary, or complex; complex roots of the form A + ib appear with their conjugates, also roots, in the same quadratic.

I. The Method

Hitchcock's method starts with an approximate quadratic divisor

$$D_0 = x^2 + p_0x + q_0$$

in the rough neighborhood of a true divisor $x^2 + \bar{p}x + \bar{q}$. Two quantities Δp and Δq are computed. (See Jour. M.P., op. cit. for actual computations and the theory of the method.) If the trial quadratic is sufficiently close to the true one, the quadratic

$$D_1 = x^2 + (p_0+\Delta p_0)x + (q_0+\Delta q_0) = x^2 + p_1x + q_1$$

is a better approximation than the first one. The process is repeated, with p_n and q_n converging to $\bar{p}$ and $\bar{q}$, respectively. Iteration is halted when the absolute values of $\Delta p/p$ and $\Delta q/q$ are as small as desired, generally 10^{-8} or 10^{-9} for these routines. The polynomial is then divided through by the quadratic and the process repeats, selecting a new trial divisor and using the final quotient as a new polynomial. Eventually, all factors are obtained.

Certain cases have been found for which Hitchcock's method does not converge. Empirically, the following conditions seem to cause the method to fail:

a. Multiplicity of the same quadratic as a factor.

b. Multiplicity of roots.

c. In some cases, closeness of roots.

d. Odd-power polynomials or those with zero roots or zero constant terms.

e. Other as yet unknown causes.

As mentioned above, an initial approximation must be supplied for each quadratic factor. These may be provided by the programmer, if he has some idea of where the roots lie, or may be generated by the routine itself. Tapes are available using each of these methods. TAPE fc141-227-62 uses the programmer's estimates; TAPE fc141-227-61 provides its own, using the three leading terms

$$a_0 + a_1x + a_2x^2$$

and the final ones

$$a_{n-2}x^{n-2} + a_{n-1}x^{n-1} + a_nx^n$$

as approximate factors.

II. Use of the Available Routines

It is recommended that the programmer run both routines. A data tape is prepared as follows:

fc141-227-30
(24,6)

al, a_n (coefficient of highest power of unknown)

a_{n-1}

a_{n-2}

.
.
.
.

a_2

a_1

a_0 (constant term)

+0.0

G.D.

Include sign and decimal point. Zero coefficients must be included.

b1, a_n }
b2, a_0 } G.D.

k1, n (degree of polynomial, single register, positive)

p1, p_0	q1, q_0	Coefficients of trial quadratics $x^2 + px + q$. If you don't know, guess.
p_1	q_1	N/2 are needed
.	.	G.D.
.	.	
.	.	
p_k	q_k	

START AT w1

The data and routine tapes must be converted together, data tape first. One run should be made with each routine tape if possible.

Results are printed out in two columns to eight decimal digits, but only the first six or seven are significant. The first column contains the p's and the second the q's of the quadratic factors. The normalized number at the end of the p column is a root sum check; the one under the column is a root product check. Each should equal one to five or more decimal places.

In some cases the routine may print out "+0.00 +0.00 +0.00", preceded by no quadratics, or fewer than expected. This means that 80 iterations with each of two trial divisors have failed to extract the next quadratic. In this case, a more accurate set of trial divisors used with tape fcl41-227-82 may produce solutions.

III. Subroutines

Because of the lack of convergence information and the unreliability of this method, preparation of a Hitchcock factorization subroutine is not recommended at present. The author hopes to continue research on this problem, and eventually some factorization subroutine may be available.

Signed: Martin Jacobs
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